Seminar

Pointing and Single Dish Amplitude Calibration Theory

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1 Introduction

In order to detect the extremely weak radio signals associated with VLBI observations, reasonably long integrations are required. However, the stronger the signal at each antenna, the more efficient will be the observations, allowing study of more sources (using less recording media) in a given time. For astronomy, not only is good sensitivity required, but the antenna also needs to be well calibrated so that the absolute radio fluxes of sources can be determined. For geodesy, the best possible signal-to-noise ratio is required. Hence, it is desirable to measure the antenna and receiver performance to assure the equipment is performing optimally or to identify areas that need to be improved. The sensitivity of an antenna depends on the aperture size and efficiency, the receiver system temperature, and whether the antenna has been properly pointed and focussed. Hence, the basic parameters of an antenna/receiver system that can be calibrated/measured are;

- The focus
- The pointing
- The aperture efficiency
- The system temperature
- The gain curve

This document introduces the basic theory of these antenna/receiver properties, their relationships and methods of measurement. I have based these notes on those of the previous TOWs by Bob Campbell (JIVE), Cormac Reynolds (then JIVE), Alastair Gunn (Jodrell Bank). Other sources of information can be found in the bibliography. More details about how these issues are dealt with practically can be found in the course notes for the maintenance workshops Automated Pointing Models Using the FS (E. Himwich), Antenna Gain Calibration (M. Lindqvist) and Generating ANTAB Tsys Files (Bach).

2 Fundamental Antenna Properties: Antenna Efficiency and Antenna Temperature

Radio antennas used in VLBI observations are normally paraboloid dishes. When such an antenna is pointed at a radio source, there will be an increase in power measured across the receiver output terminals. For a perfect antenna of area A_{eff}, the power received from an unpolarized source of flux density S (in units of Janskies $= 1 \times 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$) in a bandwidth $\Delta\nu$ is

$$P = \frac{S A_{\mathrm{eff}}}{2} \Delta\nu. \tag{1}$$

The factor of one-half results from the feed being sensitive to only one polarization of the incoming radiation. In reality, no antenna is perfect, so in practice we define the effective aperture in terms of the geometric surface area $A_{\rm geom}$ and an antenna efficiency $\eta_{\rm A}$ such that

$$A_{\rm eff} = \eta_{\rm A} A_{\rm geom} \tag{2}$$

The factor $\eta_{\rm A}$ is the fraction of incident power that is actually picked up by the receiver/antenna system. Poor focussing or pointing cause an apparent lowering of $\eta_{\rm A}$ because the antenna is not collecting as much power as it could. Generally, the antenna efficiency $\eta_{\rm A}$ can be considered as a combination of factors such as

$$\eta_{\rm A} = \eta_{\rm sf}\ \eta_{\rm bl}\ \eta_{\rm s}\ \eta_{\rm t}\ \eta_{\rm misc}, \tag{3}$$

where $\eta_{\rm sf}$ is the reflector surface-accuracy efficiency, $\eta_{\rm bl}$ is the blockage efficiency, $\eta_{\rm s}$ is the spill-over efficiency, $\eta_{\rm t}$ is the feed illumination efficiency, and $\eta_{\rm misc}$ accounts for diffraction, losses and other degrading factors. In practice, the calibration of an antenna usually involves measurement of $\eta_{\rm A}$ rather than the individual factors by which it is determined. Note that the antenna efficiency can generally be a function of elevation (see Section 6). Power at the output terminals of a receiver, when observing a radio source, can be considered equivalent to power delivered by a resistive load at some temperature T. We can then designate an antenna temperature $T_{\rm a}$ such that

$$P = kT_{\rm a}\Delta\nu, \tag{4}$$

where k is the Boltzmann constant (1.38×10^{-23} J/K). It follows that the antenna temperature of the radio source is

$$T_{\rm a} = \frac{SA_{\rm eff}}{2k}. \tag{5}$$

Larger antennas, more efficient antennas and stronger sources all give larger antenna temperatures. To find the overall antenna efficiency requires that the true antenna temperature be measured. Generally, the relative amplitudes of a radio source and a calibration signal are measured. If the calibration signal is accurately known, then $T_{\rm a}$ can be found. While it is helpful to know the absolute efficiency of an antenna, it is only the ratio of system temperature to antenna temperature that determines the sensitivity. This quantity is considered further in Section 4. In Section 6, we will discuss the sensitivity, or gain, of the telescope, which in units of Janskies per Kelvin is just $A_{\rm eff}/2k$.

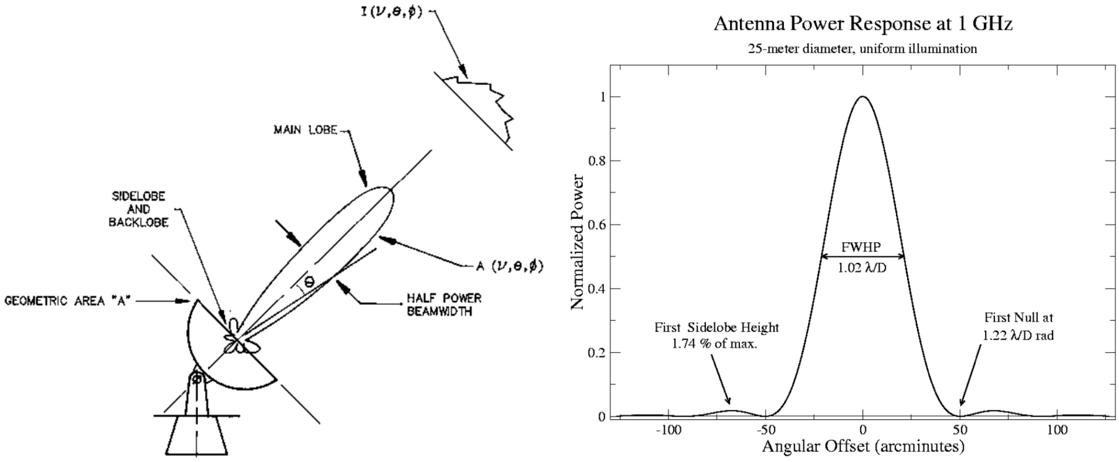


Figure 1: Antenna Response Patterns (left) and Half-Power Beamwidth of a Radio Telescope (right).

3 Fundamental Antenna Properties: Antenna Beam-width, Pointing and Focus

The range of directions over which the effective area is large is the antenna beam-width. From the laws of diffraction it can be shown that the beam-width of an antenna with characteristic size D is approximately λ/D. The radio telescope is not only sensitive to radiation propagating in the direction of the axis. For radiation incident from other directions, the relative sensitivity is given by the power pattern or beam of Gaussian form (see Figure 1).

Most sensitivity is concentrated in a smaller solid angle that is often characterized by the half-power beam-width (HPBW), which is the angle between points of the main beam where the normalized power pattern falls to 0.5 of the maximum. A source having angular size larger than the beam-width is said to be resolved, while a source of angular size much smaller than the beam-width is called an unresolved or point-like source. The HPBW is sometimes also referred to as the full-width to halfpower (FWHP). Other characterizations of the beam include beam-width between first nulls (BWFN) and the equivalent width of the main beam (EWMB). For main beams with non-circular cross-sections, values for widths in orthogonal directions are required.

Pointing calibration is fundamentally linked to the antenna response pattern (see Figure 2). Ideally, a radio source should be centered in the antenna main beam to prevent loss of signal. A pointing error of 0.1 times the HPBW causes a 3% loss in signal; for an error of 0.2 HPBW, it rises to 10% and for 0.3 HPBW it becomes 22%. Because of alignment errors, encoder offsets and deformation of the antenna, most antennas require a detailed analysis of pointing errors in order to derive a useful model for pointing corrections to within 0.1 HPBW across the entire sky. These

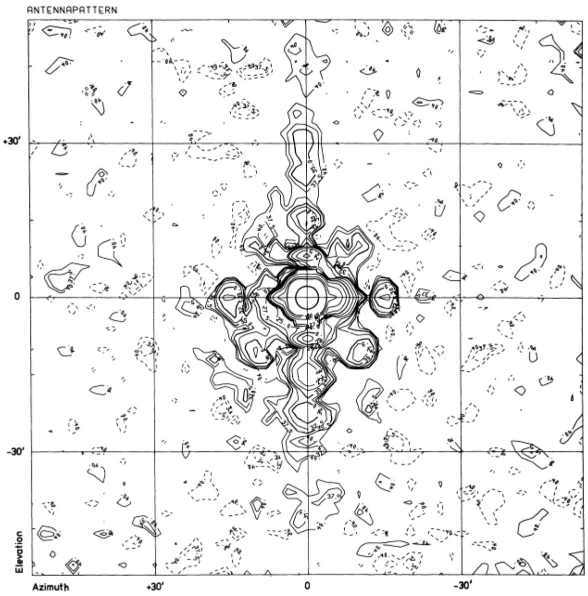


Figure 2: Primary beam pattern on the sky of the Effelsberg 100 m telescope at 11cm wavelength on 3C123. Contours are in decibel below the peak intensity in the center (Reich et al., 1978).

measurements usually consist of a set of pointing offsets for a large number of sky positions. These offsets, which are the difference between the commanded antenna position and the position at which a strong, point-like source is properly centered in the main beam, are then used in a least-squares solution to find the coefficients of the pointing equation. The offsets may be found by scanning across a source and locating the peak response, or by finding two offsets which bracket the peak in one coordinate and show equal source response. Regular checks on pointing are essential for the continued efficiency of the antenna. A full discussion of pointing calibration, and its practical determination, can be found in the maintenance workshop Automated Pointing Models Using the FS (Himwich).

The antennas used in VLBI observations are usually reflector types with a receiver feed horn located near the primary or Cassegrain focus. For optimal efficiency, the feed must be positioned exactly at the focus. Minor lateral offsets of the feed of up to a wavelength or so will mostly effect the pointing by biasing the main beam off the electrical axis, with very little loss of gain. Radial offsets in focus position, however, will significantly reduce the apparent gain of the antenna. Hence, for peak efficiency, the feed should be well within one-quarter wavelength of the radial focal point. For most antennas, the receiver box (or the Cassegrain subreflector) can be moved in and out with respect to the dish and its position should be adjusted for maximum response to a point-like source. Generally, the most accurate determination of the focus comes from locating two positions on either side of the peak response, which give the same power level, and taking the average of the two positions as the peak. However, since the radial motion of the receiver may also cause changes in spillover and standing wave patterns and thus cause output change independent of source effects, it is preferable to determine the focus by doing a series of on/off measurements on a source and using the resultant source deflections to find the correct focus. Focus should be checked at several different antenna positions since the focal length changes with the gravitational distortion of the dish. The focus should be determined at high and low elevation, then fixed at some convenient intermediate value.

4 System Noise and Source Equivalent Flux Density (SEFD)

An important fact of VLBI is that direct information about the amplitude of the received signals is lost at the stage of digitization. The correlator computes dimensionless correlation coefficients from comparing the bit-streams from each pair of telescopes. In the usual case where system noise power dominates over noise power from the source, then the net amplitude of the complex correlation coefficient is

$$C_{\mathrm{ij}} = \beta \frac{V_{\mathrm{ij}}}{\sqrt{N_{\mathrm{i}} N_{\mathrm{j}}}}, \tag{6}$$

where V_{ij} is the visibility amplitude in Jy, β is a dimensionless factor taking into account the effects of digitization, and N_{i} and N_{j} represent the system noise of the two antennas expressed as a Source Equivalent Flux Density (SEFD) in Jy.

The SEFD is defined as the source flux density which would contribute an antenna output equal to that due to the system noise, i.e. which would double the total antenna power if observed. It follows that even though C_{ij} is dimensionless we can determine V_{ij} in Jy if we know β and the SEFDs N_{i} and $_{\mathrm{j}}$ in Jy. VLBI amplitude calibration is therefore about estimating the antenna SEFD values as functions of time, elevation and frequency, and applying the resulting corrections to the raw correlation coefficients to obtain V_{ij}.

The SEFD at an antenna can be divided into two parts such that

$$N_{\mathrm{i}} = \frac{T_{\mathrm{i}}}{G_{\mathrm{i}}}, \tag{7}$$

where T_{i} is the system temperature in K and G_{i} is the antenna gain in K/Jy (you will see this sometimes denoted by Γ). The system temperature is defined as the physical temperature of a resistive load in the antenna beam that contributes the same output power as the system noise, i.e. which doubles the output power compared to having no load. The antenna gain is defined as the increase in system temperature that occurs when looking at a 1 Jy source. The SEFD therefore depends on both changes in the system temperature and in the gain. System temperatures change because of receiver variations, changes in ground spill-over, and at high frequencies changes in the atmospheric contribution.

The antenna gain G_{ij} changes mainly due to elevation dependent distortions of the dish due to gravity. It follows that the SEFD should be highly reproducible and dependent only on elevation (and perhaps HA and Dec for polar mounted telescopes). Antenna calibration can thus be divided into two halves – the system temperature calibration and the antenna gain calibration. Such absolute calibration is important for determining whether the receivers and telescope efficiencies are at their expected values, vital information for optimizing antenna performance. However, for amplitude calibration of visibilities, only the relative values of system temperature and antenna gain are necessary, i.e. SEFD. Approximate values of SEFD for EVN antennas are shown in Table 1.

Table 1: Approximate SEFD values (in K) for EVN antennas from 92cm to 0.7cm (EVN status table Apr.2013).

	wavelength [cm]										
Antenna	92	49	30	21	18	13	6	5	3.6	1.3	0.7
Jb-1 (e)	132	83		36	65		80				
Jb-2 (d)				350	320		320	300		910	
Cm (a)				220	212		136	410		720	
Wb (b)	150	90	120	30	40	60	60	1600	120		
Eb/Ef	600	600	65	20	19	300	20	25	20	90	200
Mc				490	700	400	170	840	320	700	
Nt	980s	yes	1025	820	784	770	260	1100	770	800	900
On-85			900	320	320		600	1500			
On-60						1110			1000	1380	1310
Sh					670	800	720	1500	800	1700	
Ur				300	300	560	250		350	850	
Tr			2000	250	300		220	650		500	
Mh						4500			3200	2608	
Ys							160	160	200	200	
Ar	12	12	3	3.5	3	3	5	5	6		
Wz						1250			750		
Hh					450	380	795	680	940	3000	
Sv				360	360	330	250		200	710	
Zc				300	300	330	400		200	710	
Bd				330	330	330	200		200	710	
Km						350			480		
Rob70					35	20			18	83	
Rob34						150			106		
Ny						850			1255		

System temperature, the noise in the system, is a combination of noise from various sources. Generally, we can write

$$T_{\rm sys} = T_{\rm receiver} T_{\rm ground} T_{\rm sky}, \tag{8}$$

where T (noise from the receiver system itself – LNAs, LO mixing, etc.) ranges from a few to several tens of K (for a cooled receiver), $T_{\rm ground}$ (spill-over into the sidelobes from the ground) is usually a few K, and $T_{\rm sky}$ (noise from the sky and atmosphere) depends on frequency and the water vapour content of the atmosphere. We can parameterize T_{rmsky} as

$$T_{\rm sky} = T_{\rm atm} \times (1 - e^{-\tau \sin(elv)}) + T_{\rm CMB} + T_{\rm RB}, \tag{9}$$

where $T_{\rm atm}$ (noise from the atmosphere) can be approximately 20 K to 200 K and ($T_{\rm CMB} + T_{\rm RB}$) is generally a few degrees. $T_{\rm CMB}$ is from the Cosmic Microwave Background and $T_{\rm RB}$ is from the general radio background. The τ in the exponential factor is the atmospheric opacity at zenith; the $\sin(elv)$ term is assuming a plane-parallel atmosphere above the telescope.

5 System Temperature Calibration

System temperatures can vary unpredictably during a VLBI experiment due to changes in the receiver temperature, the spill-over, RFI etc. and so must be monitored continuously. For a system with stable receiver gains, system temperatures can be monitored by recording the variation in total power from the antenna. In practice, receiver gains are insufficiently stable over long enough periods to use this method. Instead, a secondary calibration source (usually a broad-band 'noise cal' signal) of constant noise temperature $T_{\rm cal}$ is periodically injected and the change in total power is compared to the power measured when this cal signal is switched off. From these measurements the system temperature $T_{\rm sys}$ can be computed via

$$T_{\rm sys} = \frac{T_{\rm cal} P_{\rm cal-off}}{P_{\rm cal-on} - P_{\rm cal-off}}, \tag{10}$$

where $P_{\rm cal-on}$ and $P_{\rm cal-off}$ refer to the output power measured with noise cal switched on or off, and $T_{\rm cal}$ is the noise cal system temperature. Estimating $T_{\rm sys}$ obviously needs an accurate estimate of $T_{\rm cal}$. This can be measured using hot and cold loads such that

$$T_{\rm cal} = (T_{\rm hot} - T_{\rm cold}) \frac{P_{\rm cal-on} - P_{\rm cal-off}}{P_{\rm hot} - P_{\rm cold}}, \tag{11}$$

where $P_{\rm hot}$ and $P_{\rm cold}$ are the output power measured with the hot and cold loads in place, and where the measurements of noise cal and hot and cold powers are separated by a short enough time that amplifier gains do not change. The most reliable results for $T_{\rm cal}$ probably come from laboratory measurements of the

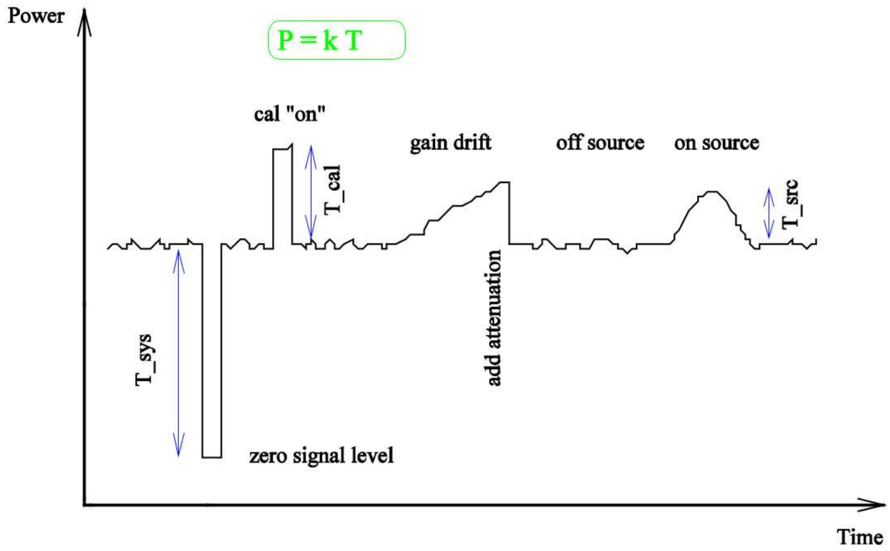


Figure 3: Power levels from a receiver in different situations.

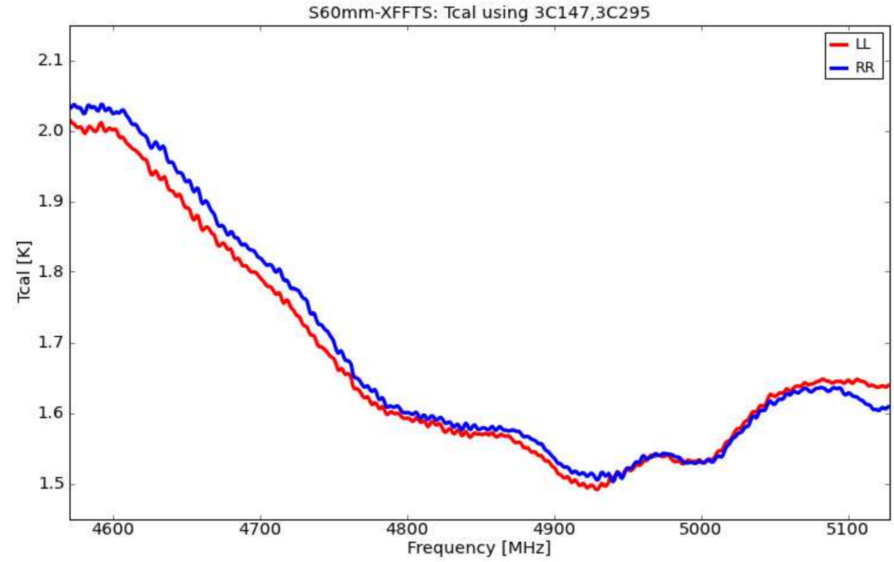
receiver and waveguide. $T_{\rm cal}$ can be a function of frequency and time, and part of normal calibration procedures is to determine these dependencies of $T_{\rm cal}$ (see Figure 4). Radio sources used for system temperature (and gain curve) calibration must necessarily be strong (tens of Janskies), of known flux density, non-variable and point-like for the antenna/receiver used. When performing calibration be sure that the radio source is not resolved (resulting in lower apparent peak flux density), that the receiver and/or power detectors are operating within the linear regime (or the VCs are not close to saturation), that the source is bright enough for reliable results and that the noise is not dominated by the atmosphere or adverse weather (possible at high frequencies) or RFI (possible at any frequency). Also ensure that power measurements are corrected for the zero level and that the pointing and focus are accurately determined.

6 Gain Curve Calibration

The gain of the antenna, expressed in K/Jy, equals the increase in total system temperature per Jansky of source flux density and depends on the collecting area of the telescope and the efficiency of the surface in focusing the incident radiation. Theoretically, this gain (in K/Jy) is proportional to the effective collecting area and equals

$$G = \frac{A\ \eta_{\rm A}}{2760}, \tag{12}$$

where A is the geometrical area in square meters and $\eta_{\rm A}$ is the aperture efficiency



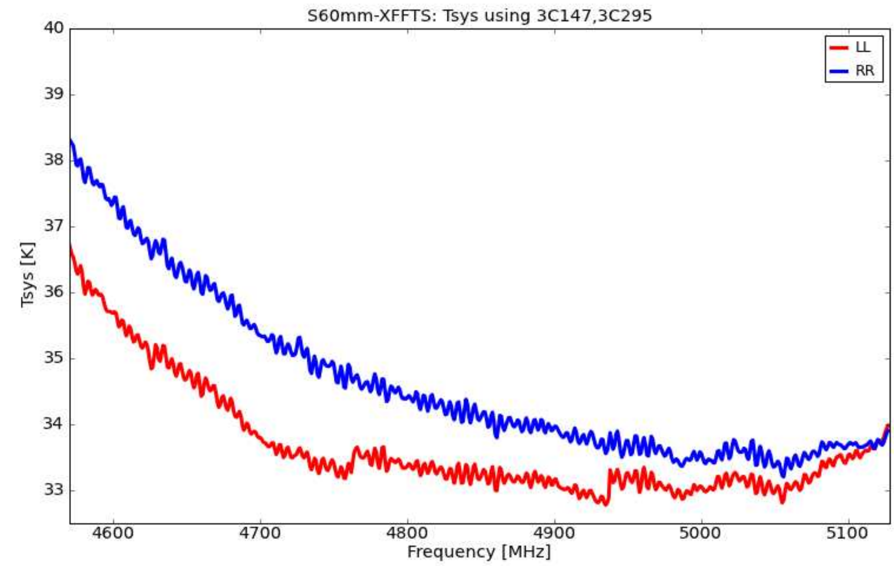


Figure 4: Examples of noise diode temperature and system temperature versus frequency for the Effelsberg 6 cm receiver. This curves are actually not derived from FS calibration but with the 32k channel FFT spectrometer.

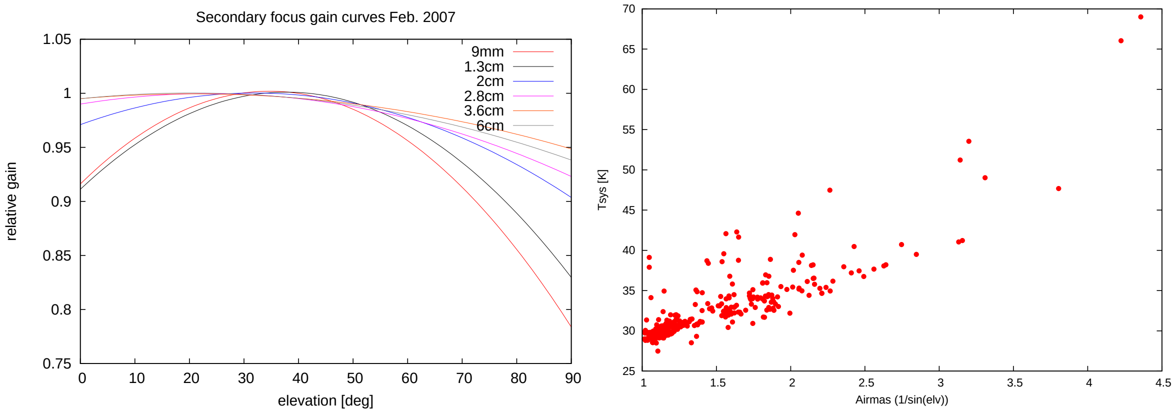


Figure 5: Examples of gain-elevation curves, $g(\mathrm{z})$, for the Effelsberg secondary focus receivers and 6cm system temperature measurements vs. airmass.

(the number in the denominator has incorporated Boltzmann's constant). For the purposes of calibration, G must be found experimentally by measuring the change in system temperature going on and off sources of known flux density. The aperture efficiency is then determined from the above equation. For most telescopes, the dominant variable factor affecting gain is the distortion due to gravity. For an altitudeazimuth mounted telescope, these gravitational effects will be solely a function of elevation. For polar mounted telescopes, this is only true to first order and more complex models may be required. The antenna gain can be parameterized in terms of an absolute gain or DPFU (Degrees Per Flux Unit) and an accompanying gain curve, g, usually expressed as a polynomial function of elevation or zenith angle z such that the DPFU multiplied by the polynomial gives the correct antenna gain at each elevation, thus

$$G(\mathrm{z}) = \mathrm{DPFU} \times g(\mathrm{z}), \tag{13}$$

where the polynomial $g(\mathrm{z})$ is

$$g(\mathrm{z}) = a_0 + a_1 \cdot \mathrm{z} + a_2 \cdot \mathrm{z}^2 + a_3 \cdot \mathrm{z}^3 + ..., \tag{14}$$

where a_{i} are the polynomial coefficients. Theoretically, the gain of the antenna can be determined by illuminating the dish with an artificial radio source of calibrated strength. However, it is then only possible to determine the gain at one elevation and it is very difficult to produce such a stable calibrated signal. More practically, the gain performance of a telescope can be found by making observations of the change in antenna temperature going on

and off celestial sources of known flux density. The gain is determined by comparing the change in power going on and off a source with the change in power when switching the noise cal on and off, such that

$$G = \frac{P_{\mathrm{on-source}} - P_{\mathrm{off-source}} T_{\mathrm{cal}}}{P_{\mathrm{cal-on}} - P_{\mathrm{cal-off}} S}, \tag{15}$$

where T_{cal} is the noise cal temperature and S is the calibrator source flux density. A convenient way to collect gain calibration data is to use the aquir program in the Field System. This program cycles around a supplied list of calibrator sources making observations of all those above the horizon. For each source, it first optionally executes the fivept task, which makes a series of observations around the nominal pointing position and fits for the position offset giving maximum power. In order to get accurate gain-elevation information, it is clearly important to have the best possible pointing. Such data can also be used to update the pointing model used during VLBI observations. Following fivept, the program onoff can be used to determine the power ratio in the above equation. Combined with estimates of the flux densities of the calibrator sources and T_{cal}, this program can be used to collect a large database of antenna gain values allowing us to fit for the gain curve $g(z)$ and the DPFU. The accuracy of the absolute gain calibration depends on the accuracy to which T_{cal} can be determined and the calibrator flux densities are known. Choice of flux calibrator is determined by the calibrator strength (some stronger sources may saturate receivers) and/or the antenna size (large antennas may partially resolve some calibrators). The details of onoff are discussed in the Antenna Gain Calibration workshop (M. Lindqvist).

7 Conclusions

Essentially, the combination of DPFU, gain curve and calibration signal temperature T_{cal} are all that are required to provide accurate calibration information for a given antenna. The absolute values of these parameters are not important, only that their combination reflects the actual performance of the antenna. How they are determined and analyzed, in general, is described in the workshops mentioned in Section 1. The acquisition of accurate antenna calibration data is very much dependent on the specific features, capabilities and priorities of individual VLBI stations, but the tools are already available within the Field System.

Bibliography

- Burke, B. F. & Graham-Smith, F., An Introduction to Radio Astronomy, 1997, Cambridge University Press – especially sections 2.2, 3.1-3.5, and 3.6
- Rohlfs, K., Tools of Radio Astronomy, 1986, Springer-Verlag: Berlin, Heidelberg – especially sections 1.3, 1.5, 1.6, 4.5[.2-.5], 5.3, 5.7, 7.2, 7.3[.1,.2,.4], and 7.7
- Kraus, J. D., Radio Astronomy, 1986, Cygnus-Quasar Books, Powell OH – especially sections 3.19, 6.2, and 7.1

- Thompson A. R., Moran J. M., Swenson G. W., Interferometry and Synthesis in Radio Astronomy, 1986, John Wiley & Sons; New York